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DEPT OF ELECTRICAL ENGINEERING
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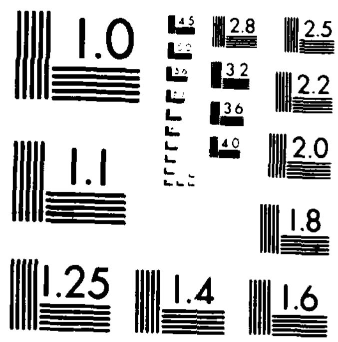
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A Technical Report
Grant No. N00014-86-K-0742
September 1, 1986 - August 31, 1988

A RANDOM ACCESS ALGORITHM FOR ENVIRONMENTS WITH CAPTURE AND LIMITED FEEDBACK CAPABILITIES

Submitted to:

Office of Naval Research
Department of the Navy
800 N. Quincy Street
Arlington, VA 22217-5000

Submitted by:

P. Papantoni-Kazakos
Professor

Daniel F. Lyons
Graduate Assistant

Report No. UVA/525415/EE88/111

March 1988

SCHOOL OF ENGINEERING AND APPLIED SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA 22901

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19. ABSTRACT

We consider synchronous packet radio systems, deploying spread-spectrum encoding. In such systems, capture events occur, where in the presence of k simultaneous packet transmissions, a single packet is captured with probability P_k. We adopt the case where $P_k = pq^{k-1}$, $k \geq 1$, for some system probabilities p and q, where ternary feedback per slot is available, where in the event of capture the identity of the captured packet is not revealed, and where capture in the presence of a single packet transmission and capture in the presence of multiple transmissions are not distinguisable events.

For the above system, we present and analyze a full sensing window random access algorithm. Due to the system model considered, the algorithm inevitably induces losses. Its performance characteristics are the fraction of lost traffic and the per successfully transmitted packet expected delays. We compute the values of those characteristics, for various values of the system and the algorithmic parameters, and for the limit Poisson user model. The algorithm can attain low delays while simultaneously inducing relatively few losses.

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I. Introduction

In some packet radio systems, severe noise conditions and/or presence of intelligent adversaries, necessitate the deployment of spread-spectrum encoding techniques. In such systems, simultaneous transmissions do not necessarily result in complete loss, and a single transmission does not necessarily result in success. Consider, for example, receiver-oriented frequency modulation spread-spectrum encoding, in an environment where additive noise and fast Rayleigh multipath fading are present. Assume also that the receiver applies per bit decoding, and consider synchronous packet transmissions with randomization of the packet starting points within a small time interval, as in [4]. Then, in the presence of k simultaneous packet transmissions, a single packet is correctly received with probability $P_k = pq^{k-1}$, where p and q are system characteristics and are probabilities. In particular, p is then a characteristic of the per packet error correcting code in conjunction with the additive noise and the Rayleigh fading parameters. The probability q is a characteristic of the randomization procedures which allow lock-in to some packet, (as in [4]), in conjunction with the propagation delays in the system.

The set $\{P_k = pq^{k-1}\}_{k \geq 1}$ defined above determines the "capture probabilties" in the system, and represents a variety of spread-spectrum systems and environments. Considering this general model of capture probabilities and the multi-user packet radio system, the issue here is the development of synchronous random-access algorithms which take possible advantage of the capture environment (we note that asynchronous algorithms are not appropriate here, since they interfere with the decoding by the receiver process). We assume the existence of ternary feedback per slot, but in contrast to the model in [2], we assume that enriched feedback capabilties are not feasible. In particular, in the event of capture a success (S) outcome is broadcasted, but the identity of the captured packet can not be recognized by any of the users in the system. Furthermore, the success in the presence of a single transmission and the capture in the presence of multiple transmission events are not distinguisable, (as in [2]).

We denote the empty, success, and collision outcomes per slot, respectfully E, S, and C, and we assume that one of those feedbacks is observed by the users at the end of each slot. We assume absence of feedback errors, and we adopt full feedback sensing. We also measure time in slot units, where slot t occupies the time interval [t, t+1). We then denote by x_t the feedback that corresponds to slot t, where x_t equals either E, or S, or C. For the above system, we propose and analyze a window random access algorithm, which is a modification of the algorithm in [1]. We note that in contrast to the algorithm in [2], the present algorithm induces losses. Indeed, due to the assumption that when capture occurs, the captured packet can not identify its own success, a portion of the input traffic is inevitably lost.

II. The Algorithm

The proposed algorithm is as this in [1], with the following modification:

Each S slot of the algorithm in [1], expands here to m+1 slots. In particular, if $x_t = S$ in the algorithm in [1], then each of the users that transmitted in slot t retransmits in one of the slots t+1,..., t+m, with probability 1/m. Just after slot t+m, all the packets that transmitted in slot t depart the system, and the algorithm in [1] reassumes its operations at time t+m+1.

For completeness, we describe here the operations of the algorithm. Let t be a time instant that corresponds to the beginning of some slot, and let t_1 be such that $t_1 < t$ and all the packet arrivals in $(0, t_1]$ have been departed at t, and there is no information regarding the arrival interval $(t_1, t]$. Then, t is a collision resolution point (CRP), and $(t_1, t]$ is the lag at t. The algorithm

utilizes a window of length Δ, in slot t the packet arrivals in $(t_1, t_2 \overset{\Delta}{=} \min(t_1+\Delta, t)]$ attempt transmission, and the arrival interval $(t_1, t_2]$ is then called the "examined interval." The examined interval is called resolved, when all the arrivals in it have departed the system, and this event is known to all users in the system. Until $(t_1, t_2]$ is resolved, no arrivals in $[t_2, \infty)$ are allowed transmission, and the time period required for the resolution of an examined interval is called collision resolution interval (CRI). The algorithmic rules are implemented independently by each user, via a counter. The counter value at time t is denoted r_t, where r_t equals either 0 or 1 or 2. The counter values are used and updated as follows:

1. A user transmits in some slot T, if and only if $r_T=1$.
2. At time t when the CRI starts, all users with arrivals in $(t_1, t_2]$ set $r_t = 1$.

(a) If $x_t = E$, then the examined interval is resolved at t, and a new CRI starts at t+1, with the examined interval $(t_2, \min(t_2+\Delta, t_2+1)]$.

(b) If $x_t = S$, then the CRI ends at t+m, and a new CRI starts at t+m+1, with the examined interval $(t_2, \min(t_2+\Delta, t_2+1+m)]$. Within the length m+1 CRI, a user who transmitted in slot t, sets: $r_{t+k} = 0; \forall k : 0 \le k \le j$, $r_{t+j+1} = 1$, with probability 1/m, where $0 \le j \le m-1$. The corresponding packet departs then the system at time t+j+1, independently on if it was successfully transmitted or not.

(c) If $x_t = C$, then at time t+1, each user who transmitted in t sets:

$$r_{t+1} = \begin{cases} 1 \,; \text{ with probability } 1/2 \\ 2 \,; \text{ with probability } 1/2 \end{cases}$$

3. Given a CRI whose first slot is a collision, let $\{t_i\}_{i \ge 1}$ be a sequence of slots in it, defined as follows: t_1 is the first slot from the beginning, such that $x_{t_1} = S$. t_{i+1} is the first slot after t_i+m, such that $x_{t_{i+1}} = S$. Then,

(a) If $r_{t_i} = 2$, then,

$r_{t_i} + j = 2, \forall j : 1 \le j \le m$

$r_{t_i + m+1} = 1$

(b) If $r_{t_i} = 1$, then,

$$\left.\begin{array}{l} r_{t_i+k} = 0 \,; \forall k{:}0 \le k \le j \\ r_{t_i+j+1} = 1 \end{array}\right\} \text{with probability } 1/m$$

and the packet departs then the system, at time t_i+j+1.

(c) If $t \ne t_i + j$, for some $1 \le j \le m$ and some t_i in $\{t_i\}_{i \ge 1}$, then:

(i) If $r_t = 2$ and $x_t = C$, then set $r_{t+1}=2$.

(ii) If $r_t = 1$ and $x_t = C$, then set:

$$r_{t+1} = \begin{cases} 1 \text{ ; with probability } 1/2 \\ 2 \text{ ; with probability } 1/2 \end{cases}$$

(iii) If $r_t = 2$ and $x_t = E$, then set $r_{t+1} = 1$.

Let us consider the sequence $\{t_i\}_{i\geq 1}$ of success slots, within a CRI, as defined in Step 3 of the algorithmic description. Then, the m slots following each t_i, are basically used by the algorithm as nonfeedback or transparent slots. We will call a pattern of m+1 consecutive slots which is headed by one of the slots in the sequence $\{t_i\}_{i\geq 1}$, an "extended-S slot." From the description of the algorithm, we easily conclude that the following possibilities exist regarding the nature of a CRI: (a) A CRI may consist of a single empty slot. (b) A CRI may consist of a single extended-S slot. (c) A CRI which begins with a collision slot ends with either two consecutive extended-S slots, or an extended-S slot followed by an empty slot, or an empty slot followed by an extended-S slot.

III. Algorithmic Analysis

Let $P_k = pq^{k-1}$, $k\geq 1$, denote the probability of capture, given k simultaneous transmissions. We will assume that in the event of capture, each of the k packets is captured with probability 1/k. Let us then define:

$0\leq n\leq k\leq 0$; $(n,k-n)$: The event that n packets have counter values equal to 1 and k-n packets have counter values equal to 2, during a CRI which starts with a collision slot.

$0\leq n\leq k\leq 0$; $L^{(m)}_{n,k-n}$: The expected number of slots needed for the resolution of the event (n,k-n), when the algorithm utilizes the integer m in steps 3.a and 3.b of its description.

$0\leq n\leq k\leq 0$; $N^{(m)}_{n,k-n}$: The expected number of lost packets during the resolution of the event (n,k-n), when the algorithm utilizes the integer m in Steps 3.a and 3.b of its operation.

$k\geq 1$; $X^{(m)}_k$: The expected number of successfully transmitted packets given k simultaneous transmissions with capture, and given that after the capture event, each of the k packets is transmitted within one of m slots, with probability 1/m.

$p^{(m)}_{c,k}$: Given that the algorithm utilizes the integer m in steps 3.a and 3.b of its operation, given k simultaneous transmissions with capture, given that after the initial capture event each of the k packets is transmitted within one of the m slots with probability 1/m, the probability that in any one of the m slots a packet is captured and this packet is different than that captured at the initial capture event.

$0\leq k$; $L^{(m)}_k$: The expected length of a CRI that starts with k simultaneous transmissions, when the algorithm utilizes the integer m in Steps 3.a and 3.b of its operation.

$0\leq k$; $N^{(m)}_k$: The expected number of lost packets throughout the length of a CRI which starts with k simultaneous transmissions, when the algorithm utilizes the integer m in Steps 3.a and 3.b of its operation.

The algorithmic rules induce then the following recursions, where w.p. means with probability.

$$\mathcal{L}_{0,0} = 1 \ , \ \mathcal{L}_{0,k} = 1+\mathrm{L}_{k,0} \ ; \ k\geq 1$$

$$\left.\begin{matrix} k\geq n\geq 1 \\ k\geq 1 \end{matrix}\right\} ; \mathcal{L}^{(m)}_{n,k-n} = \begin{cases} m+1+\mathcal{L}^{(m)}_{k-n,0} \ ; \ \text{w.p. } P_n \\ 1+\mathcal{L}^{(m)}_{i,k-i} \ ; \ \text{w.p.} \begin{bmatrix} n \\ i \end{bmatrix} 2^{-n}(1-P_n), 0\leq i\leq n \end{cases} \tag{1}$$

$$\mathcal{N}^{(m)}_{0,0} = 0 \ , \ \mathcal{N}^{(m)}_{0,k} = \mathcal{N}^{(m)}_{k,0} \ ; \ k\geq 1$$

$$\left.\begin{matrix} k\geq n\geq 1 \\ k\geq 1 \end{matrix}\right\} ; \mathcal{N}^{(m)}_{n,k-n} = \begin{cases} n-X^{(m)}_n+\mathcal{N}^{(m)}_{k-n,0} \ ; \ \text{w.p. } P_n \\ \mathcal{N}^{(m)}_{i,k-i} \ ; \ \text{w.p.} \begin{bmatrix} n \\ i \end{bmatrix} 2^{-n}(1-P_n) \ , \ 0\leq i\leq n \end{cases} \tag{2}$$

$$\left.\begin{matrix} k\geq 2 \\ m\geq 2 \end{matrix}\right\} ; p^{(m)}_{c,k} \stackrel{\Delta}{=} \frac{m-1}{m}\sum_{i=1}^{k-1}\begin{bmatrix} k-1 \\ i \end{bmatrix}\frac{1}{m^i}\left[\frac{m-1}{m}\right]^{k-1-i} P_i + \frac{1}{m}\sum_{i=1}^{k-1}\begin{bmatrix} k-1 \\ i \end{bmatrix}\frac{1}{m^i}\left[\frac{m-1}{m}\right]^{k-1-i} P_{i+1}\frac{i}{i+1} \tag{3}$$

$$X^{(0)}_k = 1 \ ; \ k\geq 1 \ , \ X^{(m)}_1 = 1 \ ; \ m\geq 0$$

$$X^{(1)}_k = 1+\frac{k-1}{k}P_k \ ; \ k\geq 1 \tag{4}$$

$$\left.\begin{matrix} m\geq 2 \\ k\geq 2 \end{matrix}\right\} ; X^{(m)}_k = 1+mp^{(m)}_{c,k}$$

$$\mathcal{N}^{(m)}_k = \mathcal{N}^{(m)}_{k,0} \ ; \ k\geq 0 \tag{5}$$

$$\mathcal{L}^{(m)}_0 = 1 \ , \ \mathcal{L}^{(m)}_k = P_k\,(1+m)+(1-P_k)\left[1+\sum_{i=0}^{k}\begin{bmatrix} k \\ i \end{bmatrix}2^{-k}\mathcal{L}^{(m)}_{i,k-i}\right] = \mathcal{L}^{(m)}_{k,0}-P_k \ ; \ k\geq 1 \tag{6}$$

Consider the algorithm in section II, and let the system start operating at time zero. Let us consider the sequence (in time) of lags induced by the algorithm, and let C_i denote the length of the i-th lag, where $i\geq 1$. Then, the first lag corresponds to the empty slot zero; thus $C_1 = 1$. In addition, the sequence $C_i, i\geq 1$, is a Markov chain whose state space is at most countable. Let D_n denote the delay experienced by the n-th successfully transmitted packet arrival, as induced by the algorithm; that is, the time between the arrival instant of the packet and the instant of its successful transmission. Let the sequence T_i, $i\geq 1$, be defined as follows: Each T_i corresponds to the beginning of some slot, and $T_1=1$. Each T_i also corresponds to the ending point of a length-one lag, and T_{i+1} is the first after T_i such point. Let R_i, $i\geq 1$, and F_i, $i\geq 1$, denote respectively the number of successfully transmitted and the number of rejected packets in the time interval $(0, T_i]$. Then $Q_i = R_{i+1} - R_i$, $i\geq 1$, and $G_i = F_{i+1} - F_i$, $i\geq 1$, denote respectively the number of successfully transmitted and the number of rejected packets in the interval $(T_i, Ti+1]$. The sequences Q_i, $i\geq 1$, and G_i, $i\geq 1$, are sequences of i.i.d. random variables, when the input traffic process is memoryless, (such as Poisson); thus the sequences R_i, $i\geq 1$, and F_i, $i\geq 1$, are then

renewal processes. In addition, the delay process D_n, $n \geq 1$, induced by the algorithm is then regenerative with respect to the process R_i, $i \geq 1$, and the process Q_i, $i \geq 1$ is nonperiodic, since $P(Q_i = 1) > 0$.

Let us consider Poisson input traffic with intensity λ. Let ρ and $\mathbf{D}$ be respectfully the fraction of successfully transmitted packets and the expected per successfully transmitted packet steady-state delay. Let us define:

$$\dot{Z} = E\{Q_1\} \ , \ W = E\{\sum_{i=1}^{Q_1} D_i\} \ , H = E\{T_2 - T_1\} \tag{7}$$

Then, from the regenerative arguments in [3], we conclude:

$$\begin{aligned} \rho &= Z(\lambda\, H)^{-1} \\ \mathbf{D} &= WZ^{-1} \end{aligned} \tag{8}$$

The pertinent quantities for the evaluation of the numbers Z, W, and H in (7), and the subsequent derivation of upper and lower bounds on ρ and $\mathbf{D}$, are given in the Appendix.

Given system probabilities p and q, we may select the algorithmic parameters m and Δ, to fulfill one of the following two objectives: Objective 1: To maximize the region of the Poisson traffic intensities, for which the expected delays of the successfully transmitted packets are finite. Objective 2: Given some lower bound ρ^* on the fraction ρ of the successfully transmitted packets, to maximize the region of the Poisson traffic intensities that satisfy this bound. Towards the fulfillment of the first objective, we optimized with respect to the window size Δ, for various values of the parameter m, and computed the expected per successfully transmitted packet delays and the fractions of the successfully transmitted packets, for various values of the Poisson traffic intensity λ. Towards the fulfillment of the second objective, we optimized with respect to the window size Δ, for various values of the parameter m and the bound ρ^*, and computed the expected per successfully transmitted packet delays, for various values of the Poisson traffic intensity λ. In all cases, we selected p=1, since the properties of our algorithm are basically exhibited by the probability (of capture, in substance) q. In the process, we computed bounds on the quantities ρ and $\mathbf{D}$, for various values of the parameters q, m, Δ, and λ. We include selective such results in Table 1, where ρ_l, ρ_u, $\mathbf{D}_l$ and $\mathbf{D}_u$, denote lower and upper bounds on ρ and lower and upper bounds on $\mathbf{D}$, respectively.

IV. Numerical Results, Conclusions and Comparisons

We present numerical results regarding the algorithm in this paper, in comparison with parallel results from a passive system. In the latter, no retransmissions are allowed. Thus, the expected per successfully transmitted packet delay equals always 1.5, while, for p=1 and for given q and Poisson traffic intensity λ, the fraction of the successfully transmitted packets is then $\lambda^{-1}\, q^{-1}\, e^{-\lambda}\, (e^{\lambda q} - 1) \stackrel{\Delta}{=} r(\lambda, q)$, where $r(\lambda, q)$ is strictly monotone with respect to λ.

In Figures 1, 2, 3, 4, and 5, we plot numerical results, when the algorithm is designed to satisfy Objective 1. In Figures 6 and 7, we plot such results when the algorithm is designed to satisfy Objective 2. In all figures, we also plot the corresponding performance of the passive system, for comparison.

In our numerical results, we selected the zero, one, and two values of the algorithmic parameter m. We note that for m=0 and q=0, the algorithm reduces to that in [1], it is then stable, and its throughput equals 0.43. From Figures 4 and 5, we observe that the m=0 case provides success rates nearly as high or higher than those in the cases of m=1 and m=2. For example, in the low capture probability case of q=0.5, the algorithm with m=0 maintains a higher success rate than the other algorithms, for all Poisson traffic rates above 0.19. Finally, it appears that selection of m=0 never penalizes the success rate substantially, and in fact, is often the best choice. From Figures 4 and 5, we also observe that the success rates induced by a passive system are significantly lower than those induced by the algorithm in this paper. From all Figures, we conclude that the m=0 selection provides the best delays, as expected. In addition, comparisons between Figures 2 and 4, and Figures 3 and 5, lead to the conclusion that the algorithm in this paper, with m=0, attains delays close to those induced by a passive system, for a significant region of Poisson intensities, while it simultaneously outperforms the latter by far, in terms of success rates.

The general conclusion drawn from our results is that the basically unmodified two-cell algorithm in [1] performs quite well in the capture environment. One may modify the algorithm as described to increase the success rate; however, a penalty is clearly paid in terms of delays. Inspection of the success rate vs. input rate plots displays immediately that there is no uniform trade-off between delays and success rate.

	λ	ρ_l	ρ_u	D_l	D_u
m=0	0.05	1.0	1.0	1.67	1.68
q=0	0.10	1.0	1.0	1.77	1.87
Δ=2.33	0.20	1.0	1.0	2.41	2.58
	0.30	1.0	1.0	4.31	4.52
	0.40	1.0	1.0	20.71	23.80
	0.10	0.975	1.000	1.69	1.70
m=0	0.20	0.945	0.991	1.89	1.96
q=0.5	0.30	0.912	0.956	2.31	2.48
Δ=2.0	0.40	0.870	0.912	3.32	3.55
	0.50	0.824	0.866	6.05	6.85
m=0	0.10	0.956	1.000	1.58	1.50
q=0.75	0.20	0.917	0.962	1.68	1.70
Δ=2.0	0.30	0.870	0.914	1.79	1.84
	0.40	0.819	0.860	1.88	2.02
m=1	0.10	1.00	1.00	2.16	2.27
q=0	0.20	1.00	1.00	4.34	4.87
Δ=3.0	0.29	1.00	1.00	45.30	58.20
m=1	0.10	0.973	1.000	1.96	1.98
q=0.5	0.20	0.942	0.990	2.71	2.85
Δ=3.0	0.30	0.848	0.943	5.20	5.75
m=1	0.10	0.967	1.000	1.72	1.74
q=0.75	0.20	0.912	0.982	1.98	2.09
Δ=3.5	0.30	0.854	0.901	2.44	2.58
	0.40	0.793	0.836	3.22	3.42
m=2	0.05	1.00	1.00	2.06	2.07
q=0	0.10	1.00	1.00	2.97	3.10
Δ=3.5	0.20	1.00	1.00	17.91	19.68
m=2	0.05	0.987	1.00	1.79	1.79
q=0.5	0.10	0.981	1.00	2.39	2.47
Δ=3.5	0.20	0.944	0.993	5.19	5.34
	0.25	0.924	0.972	9.93	11.20
m=2	0.10	0.972	1.000	2.08	2.11
q=0.75	0.20	0.926	0.981	2.88	2.99
Δ=4.0	0.30	0.868	0.919	4.32	4.49
	0.40	0.807	0.855	8.57	9.42

Table 1

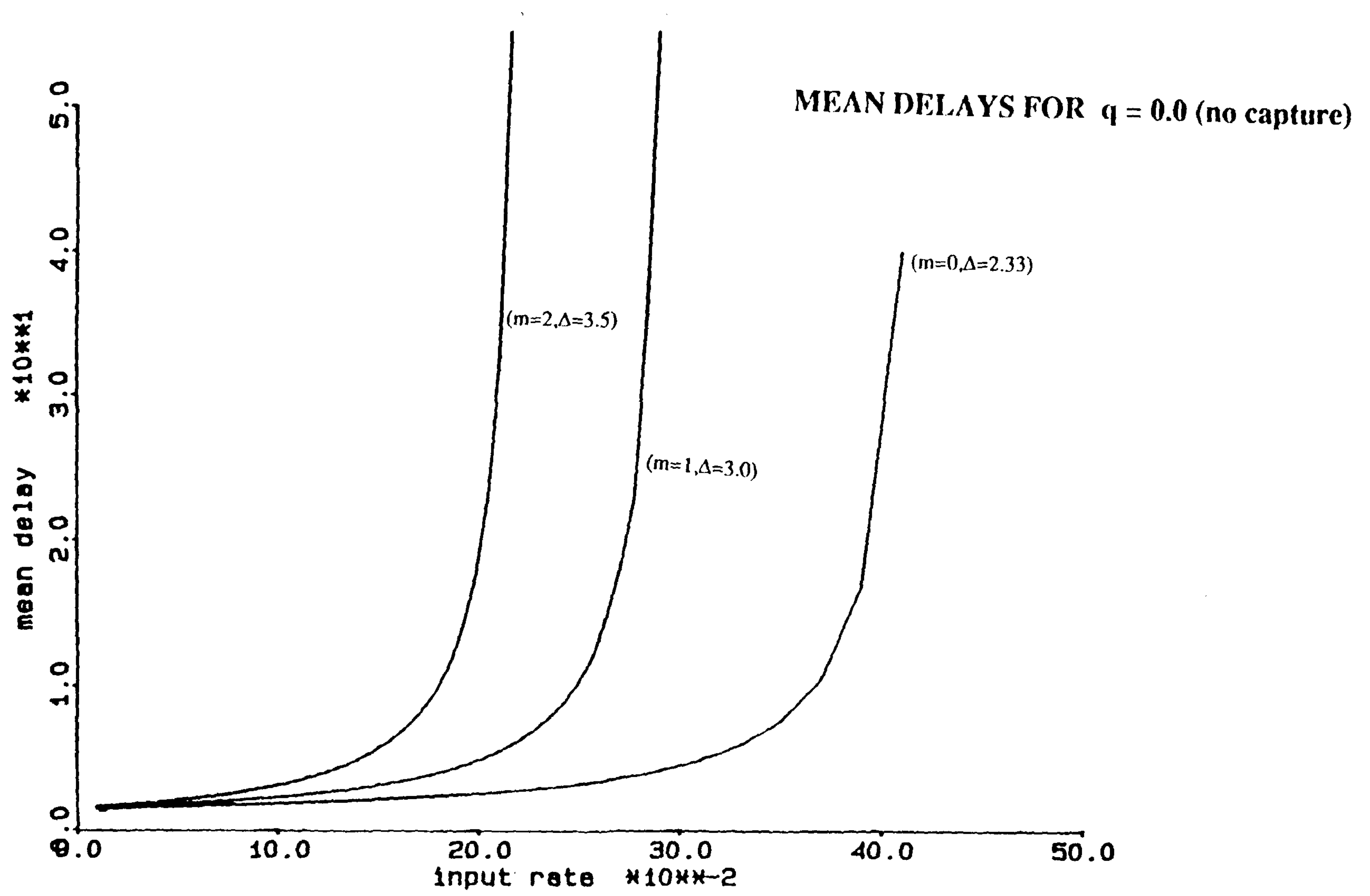

MEAN DELAYS FOR q = 0.0 (no capture)
(m=2,Δ=3.5)
(m=1,Δ=3.0)
(m=0,Δ=2.33)
mean delay ×10**1
0.0
1.0
2.0
3.0
4.0
5.0
0.0
10.0
20.0
30.0
40.0
50.0
input rate ×10**-2

Figure 1

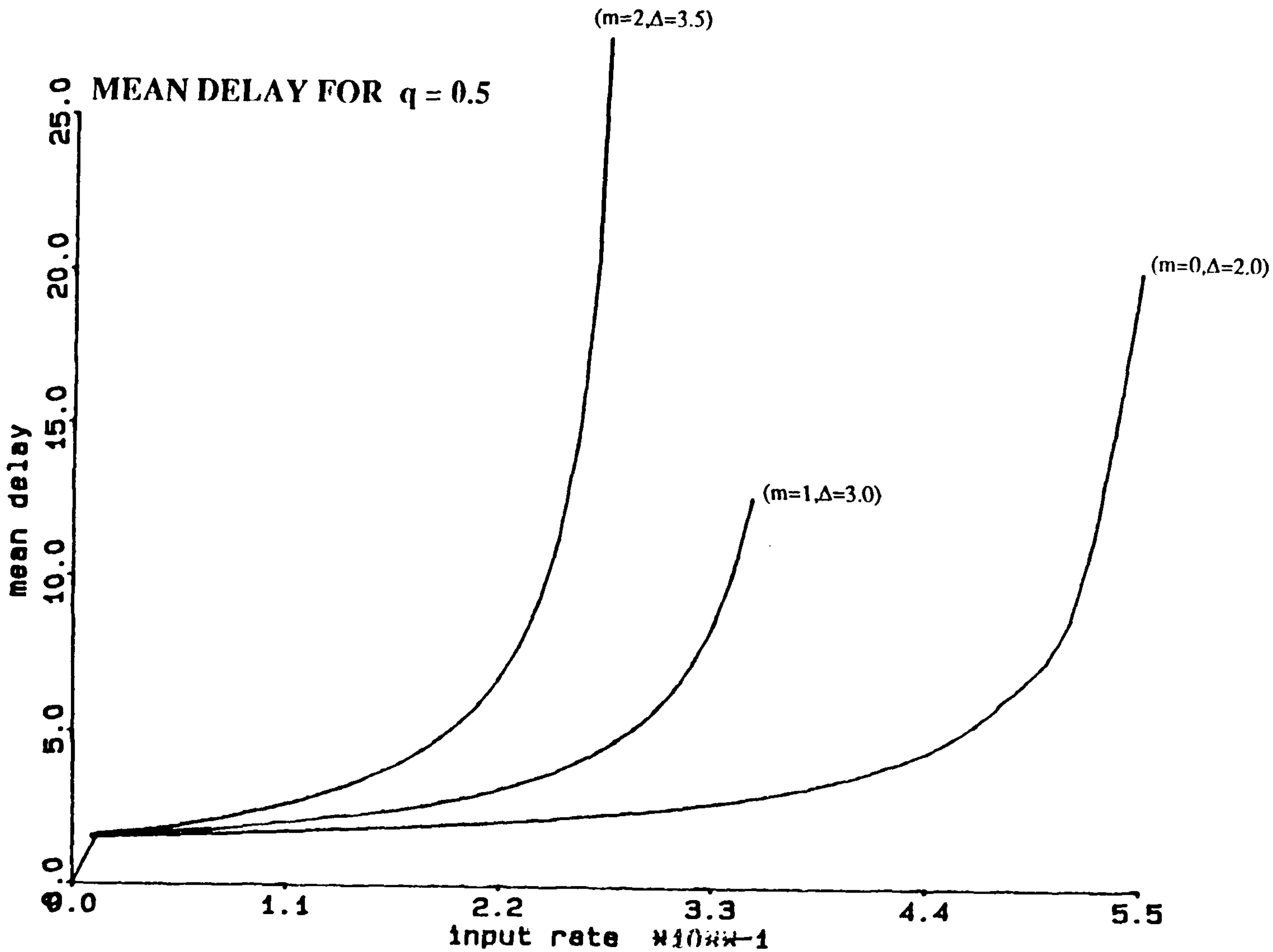

MEAN DELAY FOR q = 0.5
(m=2,Δ=3.5)
(m=1,Δ=3.0)
(m=0,Δ=2.0)
mean delay
0.0
5.0
10.0
15.0
20.0
25.0
0.0
1.1
2.2
3.3
4.4
5.5
input rate *10**-1

Figure 2

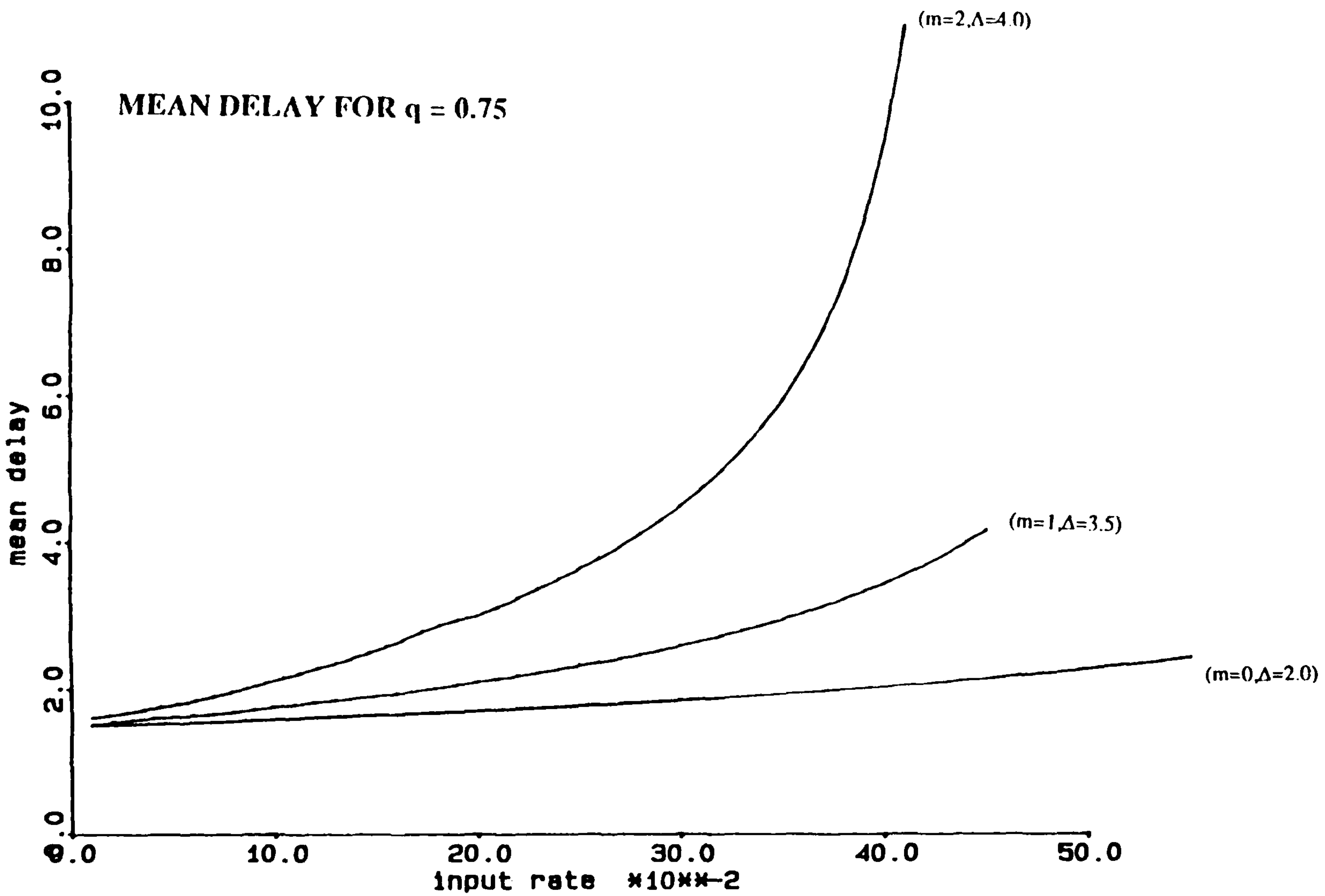

MEAN DELAY FOR q = 0.75
(m=2,Λ=4.0)
(m=1,Δ=3.5)
(m=0,Δ=2.0)
mean delay
0.0
2.0
4.0
6.0
8.0
10.0
0.0
10.0
20.0
30.0
40.0
50.0
input rate *10**-2

Figure 3

ρ_l for q = 0.5

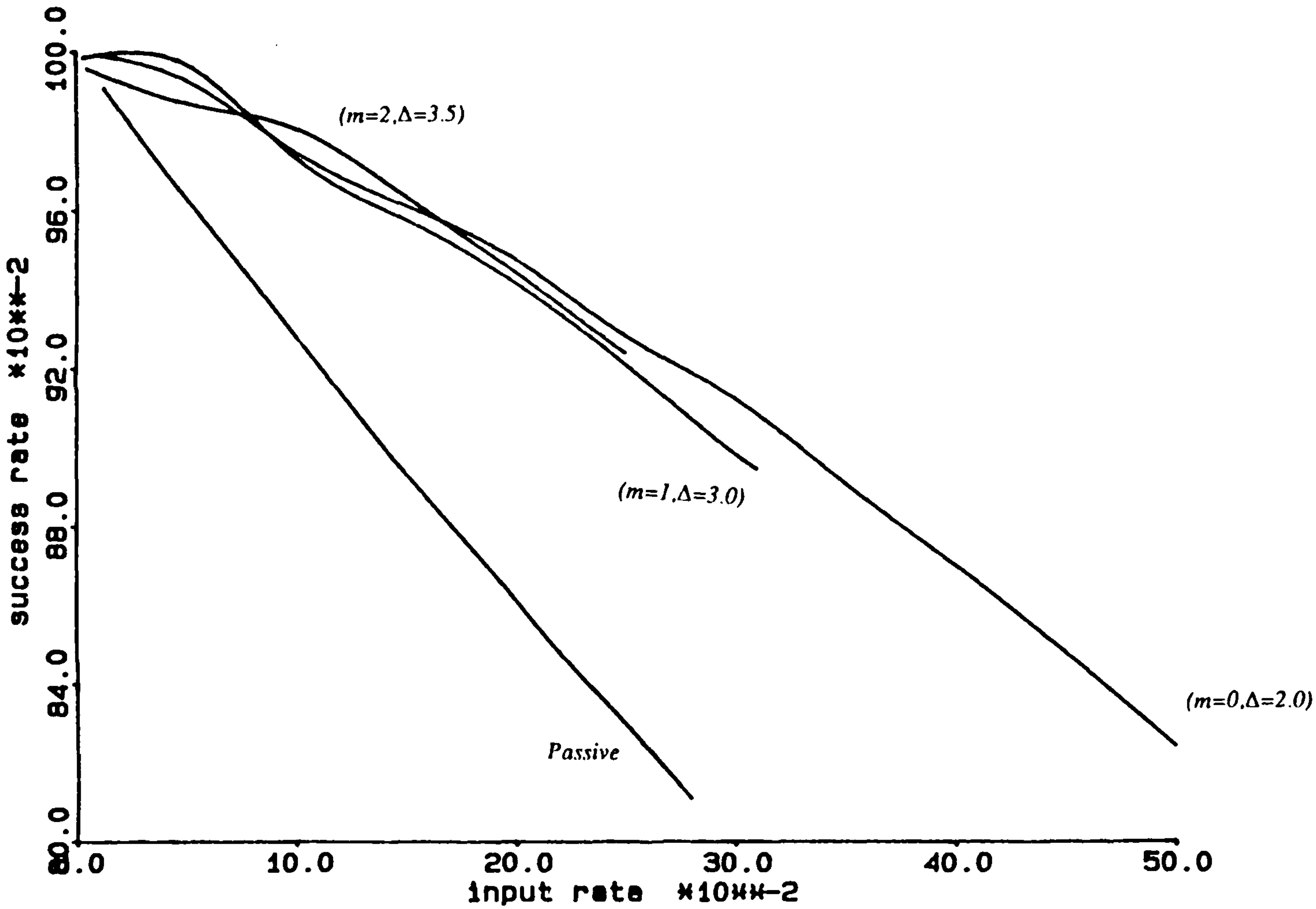


Figure 4

ρ_l for q = 0.75

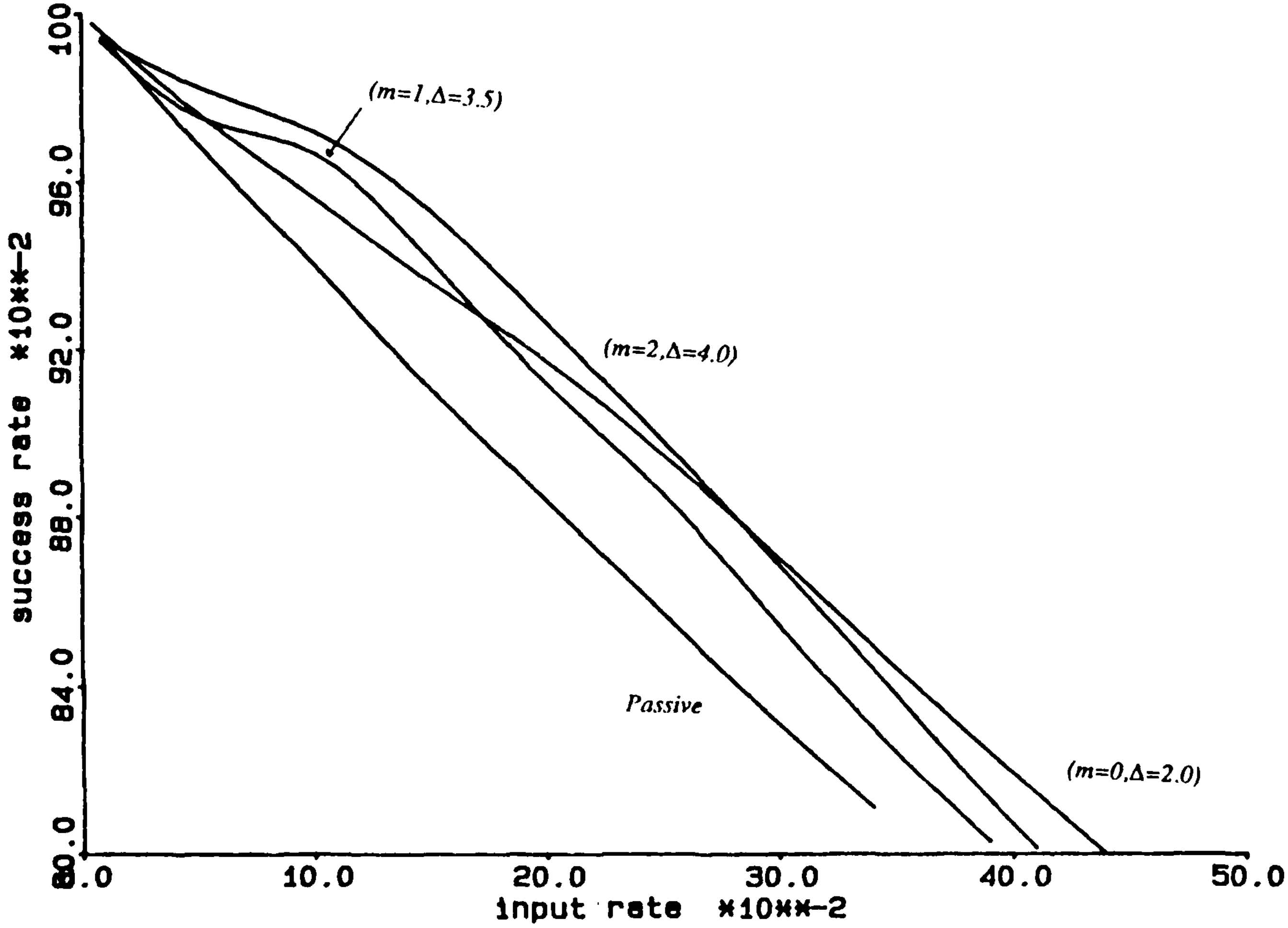
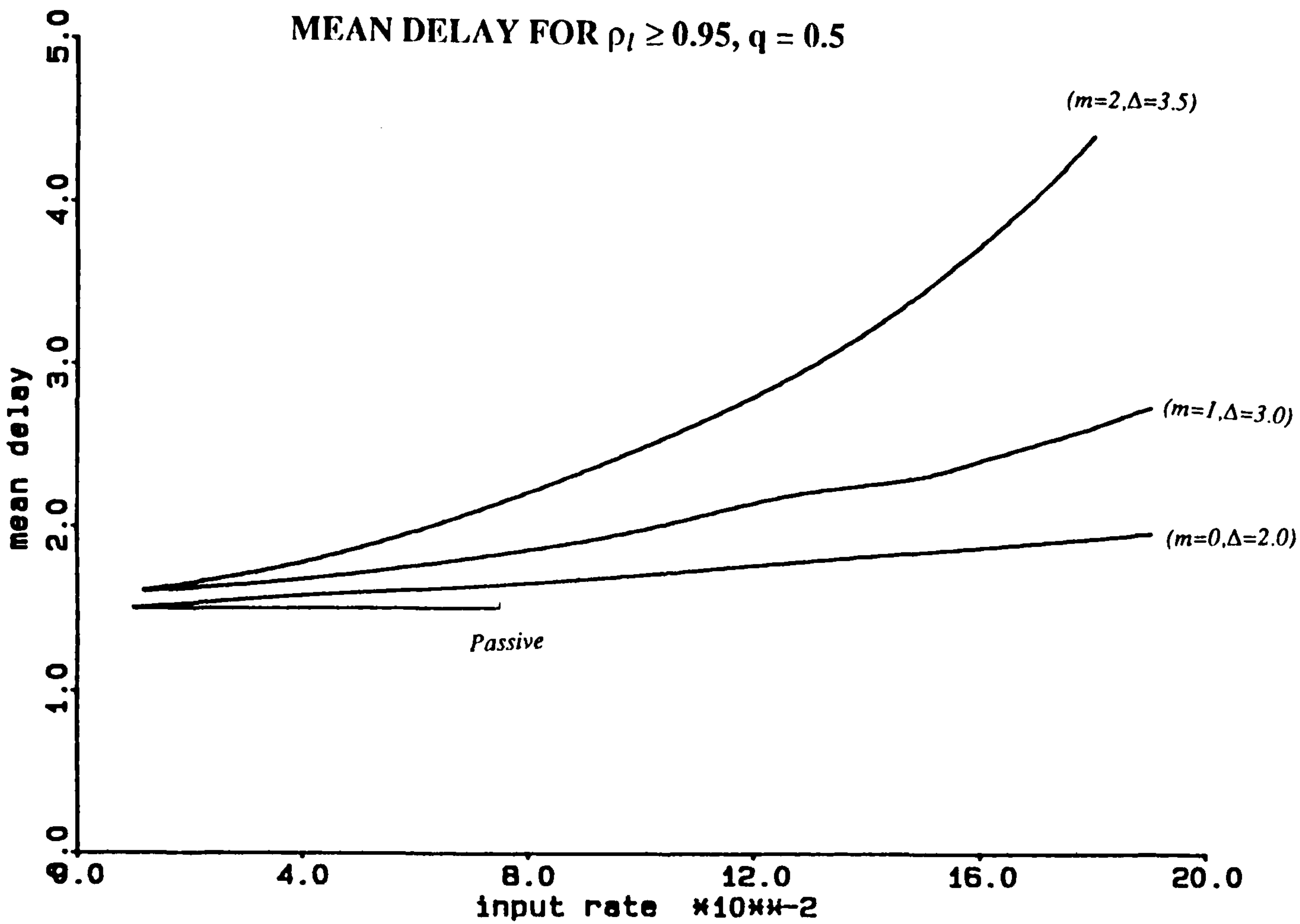


Figure 5



MEAN DELAY FOR $\rho_l \geq 0.95$, q = 0.5
mean delay
0.0
1.0
2.0
3.0
4.0
5.0
0.0
4.0
8.0
12.0
16.0
20.0
input rate *10**-2
(m=2,Δ=3.5)
(m=1,Δ=3.0)
(m=0,Δ=2.0)
Passive

Figure 6

MEAN DELAY FOR $\rho_l \geq 0.95$, q=0.75

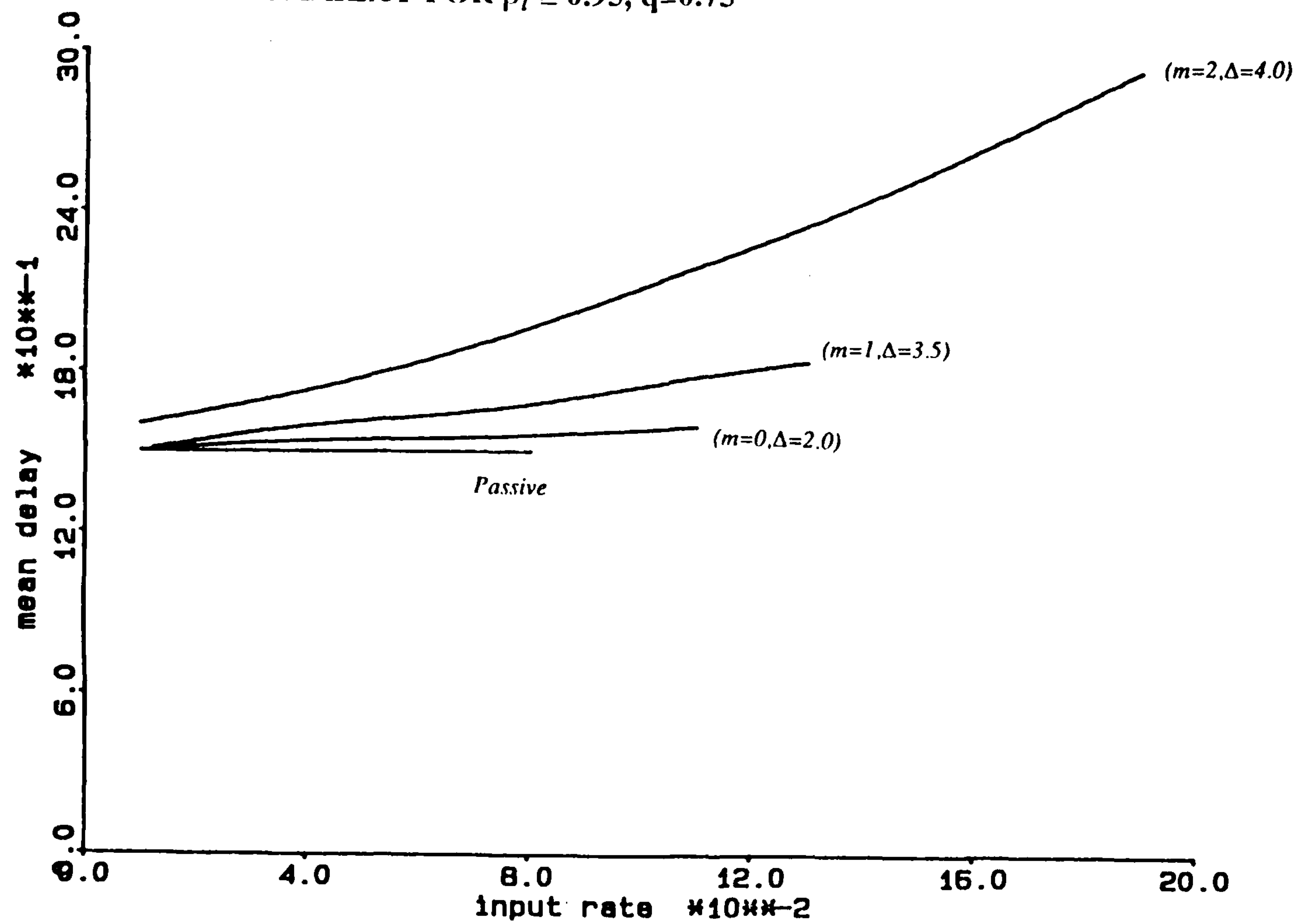


Figure 7

V. Appendix

For the computation of the expected values W, Z, and H, in (5), we need the following quantities:

n_u:	The number of packet arrivals in an examined interval of length u, that are successfully transmitted during the collision resolution process.
z_u:	The sum of delays of the n_u packets, after the beginning of the CRI.
Z_{k_1,k_2}:	Given k_i, i=1,2, packets with counter values equal to i, i=1,2, the expected sum of the delays of those that are successfully transmitted by the algorithm.
$\Psi_{u,d}$:	The sum of delays of the n_u packets, before the beginning of the CRI, given that the lag at the beginning of the CRI equals d.
l_u:	The number of slots needed to resolve an examined interval whose length is u.
h_d:	The number of slots needed to return to lag equal to one when starting from a collision resolution instant with lag d.
w_d:	The cumulative delay experienced by all the packets that were successfully transmitted during the h_d slots.
α_d:	The number of packets that are successfully transmitted within the interval that corresponds to h_d.
$P(l\mid u)$:	Given that the examined interval has length u, the probability that the corresponding collision resolution interval has length l.

$$H_d = E\{h_d\}$$

$$W_d = E\{w_d\} \tag{A.1}$$

$$A_d = E\{\alpha_d\}$$

We note that $H = H_1$, $W = W_1$, and $Z = A_1$. Also, for Poisson traffic with intensity λ, and for $L^{(m)}_{n,k-n}$ and $N^{(m)}_{n,k-n}$ as in section III, we have (for $P_k = pq^{k-1}$):

$$E\{l_u\} = \sum_{k\geq 0} e^{-\lambda u}\frac{(\lambda u)^k}{k!}L_k^{(m)} = \sum_{k\geq 1} e^{-\lambda u}\frac{(\lambda u)^k}{k!}L_{k,0}^{(m)} + \left[1+\frac{p}{q}\right]e^{-\lambda u} - \frac{p}{q}e^{-\lambda u(1-q)} \tag{A.2}$$

$$E\{n_u\} = \sum_{k\geq 0} e^{-\lambda u}\frac{(\lambda u)^k}{k!}\left[k - N_k^{(m)}\right] = \lambda u - \sum_{k\geq 1} e^{-\lambda u}\frac{(\lambda u)^k}{k!}N_k^{(m)} \tag{A.3}$$

$$E\{z_u\} = \sum_{k=0}^{\infty} e^{-\lambda u}\frac{(\lambda u)^k}{k!}Z_{k,0} \tag{A.4}$$

In addition,

$$E\{\Psi_{u,d}\} = \begin{cases} 2^{-1}u\,E\{n_u\} & ;\ 1\leq d\leq\Delta \\ \left[d - 2^{-1}u\right]E\{n_u\} & ;\ d>\Delta \end{cases} \tag{A.5}$$

The following recursions are induced by the algorithm:

$$1 \le d \le \Delta\ ;\ h_d = \begin{cases} 1\ ;\ \text{if } l_d = 1 \\ l_d + h_{l_d}\ ;\ \text{if } l_d > 1 \end{cases} \tag{A.6}$$

$$d > \Delta\ ;\ h_d = l_\Delta + h_{d-\Delta+l_\Delta}$$

$$1 \le d \le \Delta ; w_d = \begin{cases} \Psi_{d,d} + z_d\ ;\ \text{if } l_d = 1 \\ \Psi_{d,d} + z_d + w_{l_d}\ ;\ \text{if } l_d > 1 \end{cases} \tag{A.7}$$

$$d > \Delta\ ;\ w_d = \Psi_{\Delta,d} + z_\Delta + w_{d-\Delta+l_\Delta}$$

$$1 \le d \le \Delta\ ;\ \alpha_d = \begin{cases} n_d\ ;\ \text{if } l_d = 1 \\ n_d + \alpha_{l_d}\ ;\ \text{if } l_d > 1 \end{cases} \tag{A.8}$$

$$d > \Delta\ ;\ \alpha_d = n_\Delta + \alpha_{d-\Delta+l_\Delta}$$

$$k_1 \ge 1;\ Z_{k_1,k_2} = P_{k_1}\left\{ X_{k_1}^{(m)}\left[1+\frac{m+1}{2}\right] + (m+1)\left[k_2 - N_{k_2}^{(m)}\right] + Z_{k_2,0}\right.$$

$$\left. + \left(1-P_{k_1}\right)\left[k_1 + k_2 - N_{k_1,k_2}^{(m)}\right] + 2^{-k_1}\left(1-P_{k_1}\right)\sum_{i=0}^{k_1}\binom{k_1}{i} Z_{i,k_1+k_2-i} \tag{A.9}$$

$$Z_{0,k} = k - N_k^{(m)} + Z_{k,0} \tag{A.10}$$

The above recursions yield the following infinite dimensionality linear systems:

$$H_d = \begin{cases} E\{l_d\} + \sum_{l=2}^{\infty} H_l\, P(l \mid d)\ ;\ 1 \le d \le \Delta \\ E\{l_\Delta\} + \sum_{l=1}^{\infty} H_{d-\Delta+l}\, P(l \mid \Delta)\ ;\ d > \Delta \end{cases} \tag{A.11}$$

$$W_d = \begin{cases} E\{\Psi_{d,d} + z_d\} + \sum_{l=2}^{\infty} W_l P(l \mid d)\ ;\ 1 \le d \le \Delta \\ E\{\Psi_{\Delta,d} + z_\Delta\} + \sum_{l=1}^{\infty} W_{d-\Delta+l}\, P(l \mid \Delta)\ ;\ d > \Delta \end{cases} \tag{A.12}$$

$$A_d = \begin{cases} E\{n_d\} + \sum_{l=2}^{\infty} A_l \, P(l \mid d) \ ; \ 1 \le d \le \Delta \\ \\ E\{n_\Delta\} + \sum_{l=1}^{\infty} A_{d-\Delta+l} \, P(l \mid \Delta) \ ; \ d > \Delta \end{cases} \tag{A.13}$$

Let (k,n) denote the state where k packets have counter value equal to 1 and n packets have counter value equal to 2. Let $l_{k,n}^{(m)}$ denote the number of slots needed by the algorithm to go from state (k,n) to state (0,0), given that the algorithm utilizes the integer m in Steps 3.a and 3.b of its operation. Then, the following recursions are induced, where P() means probability.

$$P(l_{k,n}^{(m)}=0) = 0 \ ; \ \forall k,n \ \ , \ P(l_{0,0}^{(m)}=1) = 1 \tag{A.14}$$

$$n \ge 1 \ ; \ P\left[l_{0,n}^{(m)} = s \right] = \begin{cases} P_n \ ; \ s=m+2 \\ 2^{-n}(1-P_n) \sum_{i=0}^{n} \binom{n}{i} \ P\left(l_{1,n-i}^{(m)} = s-2\right) \ ; \ s>m+2 \end{cases} \tag{A.15}$$

$$\left.\begin{matrix} k \ge 1 \\ n \ge 0 \end{matrix}\right\} \ ; \ P\left[l_{k,n}^{(m)} = s \right] = \begin{cases} P_k \ ; \ s=m+1 \\ 2^{-k}(1-P_k) \sum_{i=0}^{k} \binom{k}{i} \ P\left[l_{1,k+n-i}^{(m)} = s-1 \right] \ ; \ s>m+1 \end{cases} \tag{A.16}$$

In addition, given Poisson traffic intensity λ, we have:

$$P(l \mid u) = \sum_{k=0}^{\infty} e^{-\lambda u} \frac{(\lambda u)^k}{k!} \ P(l_{k,0}^{(m)} = l) \tag{A.17}$$

Bounds

Let us define the sets $\{B_n\}_{n\ge1}$, $\{A_n^{(j)}, 0\le j\le n\}_{n\ge1}$, $\{C_n\}_{n\ge1}$, and $\{D_n^{(j)}, 0\le j\le n\}_{n\ge1}$, as follows:

$$B_1 \stackrel{\Delta}{=} (1+P_1)^{-1}[3+(2m-1)P_1] \tag{A.18}$$

$$B_n \stackrel{\Delta}{=} [1-2^{-n}(1-P_n)]^{-1}\left\{ 1+mP_n+2^{-n}(1-P_n)[1+\sum_{i=1}^{n-1}\binom{n}{i} B_i \right\} \ , \ n\ge2$$

$$A_1^{(0)} \stackrel{\Delta}{=} (1+P_1)^{-1}(1-P_1)$$

$$A_n^{(0)} \stackrel{\Delta}{=} [1-2^{-n}(1-P_n)]^{-1} 2^{-n}(1-P_n)\left[1+\sum_{i=1}^{n-1}\binom{n}{i} A_i^{(0)} \right] \ , \ n\ge2 \tag{A.19}$$

$$A_n^{(j)} \triangleq [1-2^{-n}(1-P_n)]^{-1}\, 2^{-n}(1-P_n) \sum_{i=j}^{n-1} \begin{bmatrix} n \\ i \end{bmatrix} A_i^{(j)} \;,\; 1\le j\le n-1, n\ge 2$$

$$A_n^{(n)} \triangleq [1-2^{-n}(1-P_n)]^{-1}\, P_n \;,\; n\ge 1$$

$$C_1 \triangleq 0 \;,\; C_2 \triangleq (1+P_2)^{-1}\, 2P_2[2-X_2^{(m)}] \tag{A.20}$$

$$C_n \triangleq [1-2^{n}(1-P_n)]^{-1} \left\{ P_n[n-X_n^{(m)}] + 2^{-n}(1-P_n) \sum_{i=2}^{n-1} \begin{bmatrix} n \\ i \end{bmatrix} C_i \right\} ,\; n\ge 3$$

$$D_1^{(0)} \triangleq (1+P_1)^{-1}\,(1-P_1)$$

$$D_n^{(0)} \triangleq [1-2^{-n}(1-P_n)]^{-1}\, 2^{-n}(1-P_n) \left[1 + \sum_{i=1}^{n-1} \begin{bmatrix} n \\ i \end{bmatrix} D_i^{(0)} \right] ,\; n\ge 2 \tag{A.21}$$

$$D_n^{(j)} \triangleq [1-2^{-n}(1-P_n)]^{-1}\, 2^{-n}(1-P_n) \sum_{i=j}^{n-1} \begin{bmatrix} n \\ i \end{bmatrix} D_i^{(j)} \;,\; 1\le j\le n-1 \;,\; n\ge 2$$

$$D_n^{(n)} \triangleq [1-2^{-n}(1-P_n)]^{-1} P_n$$

It can be found by induction, that there exist natural numbers n_0 and k_0, such that:

$$b_l + a_l\, n < B_n < b_u + a_u\, n \qquad ;\ \forall\, n > n_0 \tag{A.22}$$

$$c_l^{(j)} < A_n^{(j)} < c_u^{(j)} \;,\; 0\le j\le n$$

$$d_l < C_n < d_u \qquad ;\ \forall\, n > k_0 \tag{A.23}$$

$$e_l^{(j)} < D_n^{(j)} < e_u^{(j)} \;,\; 0\le j\le n$$

It is also found then, that:

$$L_{n,k-n}^{(m)} = B_n + \sum_{i=0}^{n} A_n^{(i)}\, L_{k-i,0}^{(m)}$$

and thus,

$$L_{1,0}^{(m)} = [1-A_1^{(0)}]^{-1}\, [B_1 + A_1^{(1)}] \tag{A.24}$$

$$L^{(m)}_{k,0} = [1-A^{(0)}_k]^{-1}\left\{B_k + A^{(k)}_k + \sum_{i=1}^{k-1} A^{(i)}_k\, L^{(m)}_{k-i,0}\right\}$$

$$N^{(m)}_{n,k-n} = C_n + \sum_{i=0}^{n} D^{(i)}_n\, N^{(m)}_{k-i,0}$$

and thus,

$$N^{(m)}_{1,0} = 0\ ,\ N^{(m)}_{2,0} = (P_1+P_2)^{-1}\, P_2(1+P_1)[2-X^{(m)}_2]$$

$$N^{(m)}_{k,0} = [1-D^{(0)}_k]^{-1}\left\{C_k + \sum_{i=1}^{k-2} D^{(i)}_k\, N^{(m)}_{k-i,0}\right\}\ ,\ k\geq 3 \tag{A.25}$$

From (A.21) in conjunction with (A.23), and from (A.26) in conjunction with (A.24), we conclude that n_0 and k_0 can be found, such that there exist constants A_u, A_l, B_u, B_l, C_u, C_l, D_u,D_l, F_u, F_l which satisfy the inequalities:

$$C_l + B_l\, k + A_l\, k^2 < L^{(m)}_{k,0} < A_u\, k^2 + B_u\, k + C_u\ ;\ \forall k>n_0 \tag{A.27}$$

$$F_l + D_l\, k < N^{(m)}_{k,0} < D_u\, k + F_u\ ;\ \forall k>k_0 \tag{A.28}$$

and the bounds in (A.27) and (A.28) are tight. We used those bounds to bound the quantities in (A.2) and (A.3).

Let us define,

$$E_n \overset{\Delta}{=} [1-2^{-n}(1-P_n)]^{-1}\left\{P_n[X^{(m)}_n(1+m/2)-m/2] + 2^{-n}(1-P_n)\sum_{i=0}^{n-1}\binom{n}{i} E_i\right\}\ ;\ n\geq 1 \tag{A.29}$$

$$F^{(j)}_n \overset{\Delta}{=} \begin{cases} P_n\,[1-2^{-n}(1-P_n)]^{-1} & ;\ j=n \\ [1-2^{-n}(1-P_n)]^{-1}\, 2^{-n}(1-P_n)\displaystyle\sum_{i=j}^{n-1}\binom{n}{i} F^{(j)}_i & ;\ 0\leq j\leq n-1 \end{cases} \tag{A.30}$$

$$G^{(j)}_n \overset{\Delta}{=} \begin{cases} (m+1)P_n\,[1-2^{-n}\,(1-P_n)]^{-1} & ;\ j=n \\ [1-2^{-n}(1-P_n)]^{-1}\, 2^{-n}(1-P_n)\displaystyle\sum_{i=j}^{n-1}\binom{n}{i} G^{(j)}_i & ;\ 0\leq j\leq n-1 \end{cases} \tag{A.31}$$

Then, from (A.9), (A.29), (A.30), and (A.31), we find:

$$Z_{n,k-n} = E_n + \sum_{i=0}^{n-1} F^{(i)}_n\, Z_{k-i,0} + \sum_{j=0}^{n-1} G^{(j)}_n\,[k-N_{k-i,0}] \tag{A.32}$$

Also, from (A.29) - (A.31), and by induction, we find that there exist constants f_l, e_l, f_u, e_u, g_l, g_u, h_l, and h_u, and some n_0, such that:

$$f_l + e_l\, n \le E_n \le f_u + e_u\, n \;\; ; \forall\, n \ge n_0$$

$$g_l \le F_n^{(j)} \le g_u \; ; 0 \le j \le n \, , \; \forall\, n \ge n_0 \tag{A.33}$$

$$h_l \le G_n^{(j)} \le h_u \; ; 0 \le j \le n, \; \forall\, n \ge n_0$$

Substituting the bounds in (A.33), in expression (A.32), we find:

$$f_l + (g_l + h_l)\, k + e_l\, k^2 \le Z_{k,0} \le f_u + (g_u + h_u)\, k + e_u\, k^2 \; ; \forall k \ge n_0 \tag{A.34}$$

We used the bounds in (A.34) to compute bounds on the expected value in (A.4).

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